Entropy of Kaluza-Klein Black Hole from Kerr/CFT Correspondence

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Abstract

We extend the recently proposed Kerr/CFT correspondence to examine the dual conformal field theory of four dimensional Kaluza-Klein black hole in Einstein-Maxwell-Dilaton theory. For the extremal Kaluza-Klein black hole, the central charge and temperature of the dual conformal field are calculated following the approach of Guica, Hartman, Song and Strominger. Meanwhile, we show that the microscopic entropy given by the Cardy formula agrees with Bekenstein-Hawking entropy of extremal Kaluza-Klein black hole. For the non-extremal case, by studying the near-region wave equation of a neutral massless scalar field, we investigate the hidden conformal symmetry of Kaluza-Klein black hole, and find the left and right temperatures of the dual conformal field theory. Furthermore, we find that the entropy of non-extremal Kaluza-Klein black hole is reproduced by Cardy formula.

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I. INTRODUCTION

The recently proposed Kerr/CFT correspondence [1] states that quantum gravity in the region very near the horizon of an extreme Kerr black hole with proper boundary conditions is holographically dual to a two-dimensional chiral conformal field theory with the central charge proportional to angular momentum. It is shown that the macroscopic Bekenstein-Hawking entropy of extremal Kerr black hole can be reproduced by the microscopic entropy of dual conformal field theory via Cardy formula. The method employed by Guica, Hartman, Song and Strominger(GHSS) in [1] is very similar to the approach of Brown and Henneaux in [2], where the AdS_3 background is replaced by the near-horizon extremal Kerr(NHEK) geometry previously obtained in [3]. This method has been generalized to calculate the entropies of extremal black holes in a lot of theories such as the Einstein theory, string theory, and supergravity theory, as well as those solutions in diverse dimensions[5–39].

Very recently, Castro, Maloney and Strominger (CMS) in a remarkable paper [40] show that there exists a hidden conformal symmetry for four dimensional non-extremal Kerr Black hole by studying the near-region wave equation of a massless scalar field. Interestingly, the hidden conformal symmetry is not derived from the conformal symmetry of spacetime geometry itself. It is also shown that microscopic entropy computed by Cardy formula agrees exactly with the macroscopic Berenstein-Hawking entropy of non-extremal Kerr black hole. These observations suggest that non-extremal Kerr black hole is holographically dual to a two-dimensional conformal field theory with non-zero left and right temperatures. Subsequently, with the idea of hidden conformal symmetry of non-extremal black holes, Krishnan [41] extended this method to investigate five-dimensional black holes in string theory. Furthermore, the non-extremal uplifted 5D Reissner-Nordstrom black hole was investigated by Chen and Sun [42], and the hidden conformal symmetry of Kerr-Newman black hole was studied by Wang et al. in [43] and Chen et al. in [44].

In this paper, We extend the Kerr/CFT correspondence to examine the dual conformal field theory of four dimensional Kaluza-Klein black hole in Einstein-Maxwell-Dilaton theory. For the extremal Kaluza-Klein black hole, we firstly perform a coordinates transformation to find the near horizon extremal geometry. Then, by employing the approach of GHSS, the central charge and the left and right temperatures of dual conformal field are calculated. Finally, we find the microscopic entropy calculated by using Cardy formula agrees with

Bekenstein-Hawking entropy of the extremal Kaluza-Klein black hole. For the non-extremal case, we investigate the hidden conformal symmetry of Kaluza-Klein black hole by studying the near-region wave equation of a neutral massless scalar field in this background, and find the left and right temperatures of dual conformal field theory. Furthermore, the entropy of non-extremal Kaluza-Klein black hole is reproduced by using Cardy formula.

It should be noted that the holographic dual of the extremal uplifted 5D Kaluza-Klein black hole has been investigated by Azeyanagi et.al. in [6]. They show that the central charge of dual conformal field is $c = 12J$, where J is the angular momentum of 5D Kaluza-Klein black hole. In this paper, we treat the Kaluza-Klein black hole as a exact solution of Einstein-Maxwell-Dilaton theory. It seems that there exists a non-vanishing contributions to the central charge from gauge field and Dilaton field. Fortunately, an explicit calculation given by Compere et al. in [7] shows that the central charge receives no contribution from the non-gravitational fields, i.e. only the Einstein-Hilbert Lagrangian contributes to the value of the central charge. So, it is sufficient to only calculate the gravitational field contribution to the central charge for four dimensional Kaluza-Klein black hole in the present situation. Our result presented in Sec.III agrees with the observation in [6], which confirms this viewpoint.

This paper is organized as follows. In section II, we give a brief review of four dimensional Kaluza-Klein black hole. In section III, we calculate the central charge and the left and right temperatures of the dual conformal field theory for the extremal Kaluza-Klein black hole, and find the microscopic entropy of the dual CFT. In section IV, we study the hidden conformal symmetry of the non-extremal Kaluza-Klein black hole by analysing the near-region wave equation of a neutral massless scalar field. Furthermore, the microscopic entropy of dual CFT with non-zero left and right temperatures are obtained. The last section is devoted to discussion.

II.   KALUZA-KLEIN BLACK HOLE

In this section, we will give a brief review of four dimensional Kaluza-Klein black hole. Kaluza-Klein black hole solution is derived by a dimensional reduction of the boosted five-dimensional Kerr solution to four dimensions. It is also an exact solution of Einstein-

Maxwell-Dilaton action. The metric is explicitly given by[45, 46]

$$ds^2 = -\frac{1-Z}{B}dt^2 - \frac{2aZ\sin^2\theta}{B\sqrt{1-\nu^2}}dtd\varphi + \frac{B\Sigma}{\Delta}dr^2$$
$$+ \left[B(r^2+a^2) + a^2\sin^2\theta\frac{Z}{B}\right]\sin^2\theta d\varphi^2 + B\Sigma d\theta^2 \,, \tag{1}$$

where

$$Z = \frac{2\mu r}{\Sigma} \,,$$
$$B = \sqrt{1 + \frac{\nu^2 Z}{1-\nu^2}} \,,$$
$$\Sigma = r^2 + a^2\cos^2\theta \,,$$
$$\Delta = r^2 - 2\mu r + a^2 \,. \tag{2}$$

The dilaton field and gauge potential are given by respectively

$$\phi = -\frac{\sqrt{3}}{2}\ln B \,,$$
$$A = \frac{\nu}{2(1-\nu^2)}\frac{Z}{B^2}dt - \frac{a\nu\sin^2\theta}{2\sqrt{1-\nu^2}}\frac{Z}{B^2}d\varphi \,. \tag{3}$$

The physical mass M, the charge Q, and the angular momentum J are expressed with the boost parameter ν, mass parameter μ, and specific angular momentum a as

$$M = \mu\left[1 + \frac{\nu^2}{2(1-\nu^2)}\right] \,,$$
$$Q = \frac{\mu\nu}{1-\nu^2} \,,$$
$$J = \frac{\mu a}{\sqrt{1-\nu^2}} \,. \tag{4}$$

The outer and inner horizons are respectively given by

$$r_\pm = \mu \pm \sqrt{\mu^2 - a^2} \,. \tag{5}$$

Hawking temperature, entropy and angular velocity of the event horizon are respectively given by

$$T_H = \frac{\sqrt{1-\nu^2}}{2\pi}\frac{\mu^2 - a^2}{r_+^2 + a^2} \,,$$
$$\Omega_H = \frac{a\sqrt{1-\nu^2}}{r_+^2 + a^2} \,,$$
$$S = 2\pi\frac{\mu}{\sqrt{1-\nu^2}}(\mu + \sqrt{\mu^2 - a^2}) \,. \tag{6}$$

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The extremality condition is $\mu = a$, and the entropy at extremality is

$$S(T_H = 0) = \frac{2\pi\mu^2}{\sqrt{1 - \nu^2}} \ . \tag{7}$$

In the following two sections, we will try to reproduce the Bekenstein-Hawking entropies of the extremal and non-extremal Kaluza-Klein black hole via Cardy formula of the dual conformal field.

III. ENTROPY OF EXTREMAL KALUZA-KLEIN BLACK HOLE FROM KERR/CFT DUAL

In this section, our purpose is to derive Bekenstein-Hawking entropy of extremal Kaluza-Klein black hole via the extremal Kerr/CFT correspondence.

We now try to explore the near-horizon geometry of extremal Kaluza-Klein black hole. To do so, we need to perform the following coordinate transformations

$$
\begin{aligned}
r &= a + \epsilon r_0 \hat{r} \ , \\
t &= \frac{r_0 \hat{t}}{\epsilon} \ , \\
\varphi &= \hat{\varphi} + \frac{\sqrt{1 - \nu^2}}{2a} \frac{r_0 \hat{t}}{\epsilon} \ ,
\end{aligned}
\tag{8}
$$

with the parameter $r_0^2 = \frac{2a^2}{\sqrt{1-\nu^2}}$. After taking the $\epsilon \to 0$ limit, the near-horizon geometry of an extremal Kaluza-Klein black hole reads

$$
\begin{aligned}
ds^2 &= \hat{B}a^2(1 + \cos^2\theta)\left(-\hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + d\theta^2\right) \\
&+ \frac{4a^2}{\hat{B}(1 - \nu^2)} \frac{\sin^2\theta}{1 + \cos^2\theta}\left(d\hat{\varphi} + \hat{r}d\hat{t}\right)^2 \ ,
\end{aligned}
\tag{9}
$$

with

$$\hat{B} = \left[1 + \frac{2\nu^2}{(1 - \nu^2)(1 + \cos^2\theta)}\right]^{\frac{1}{2}} \ . \tag{10}$$

We now employ the Brown-Henneaux approach to find the central charge of the dual holographic conformal field theory description of an extremal Kaluza-Klein black hole. As explained in the introduction, the calculation carried by Compere et al in [7] indicates that, in Einstein-Maxwell-Dilaton theory with topological terms in four and five dimensions, the central charge receives no contribution from the non-gravitational fields. To find the central

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charge of the dual conformal field for four dimensional Kaluza-Klein black hole in Einstein-Maxwell-Dilaton theory, for simplicity, it is sufficient to only calculate the gravitational field contribution.

It is important to impose the appropriate boundary conditions at spatial infinity and find the asymptotical symmetry group that preserves these boundary conditions. We choose the boundary conditions

$$
\begin{pmatrix}
h_{\hat{t}\hat{t}} = \mathcal{O}(\hat{r}^2) \;\; h_{\hat{t}\hat{\varphi}} = \mathcal{O}(1) \;\; h_{\hat{t}\theta} = \mathcal{O}(\frac{1}{\hat{r}}) \;\; h_{\hat{t}\hat{r}} = \mathcal{O}(\frac{1}{\hat{r}^2}) \\
h_{\hat{\varphi}\hat{\varphi}} = \mathcal{O}(1) \;\; h_{\hat{\varphi}\theta} = \mathcal{O}(\frac{1}{\hat{r}}) \;\; h_{\hat{\varphi}\hat{r}} = \mathcal{O}(\frac{1}{\hat{r}}) \\
h_{\theta\theta} = \mathcal{O}(\frac{1}{\hat{r}}) \;\; h_{\theta\hat{r}} = \mathcal{O}(\frac{1}{\hat{r}^2}) \\
h_{\hat{r}\hat{r}} = \mathcal{O}(\frac{1}{\hat{r}^3})
\end{pmatrix}
\tag{11}
$$

where $h_{\mu\nu}$ is the metric deviation from the near horizon geometry. The diffeomorphism symmetry that preserves such a boundary condition is generated by the vector field

$$
\zeta = \epsilon(\hat{\varphi})\frac{\partial}{\partial\hat{\varphi}} - \hat{r}\epsilon'(\hat{\varphi})\frac{\partial}{\partial\hat{r}} \;,
\tag{12}
$$

where $\epsilon(\hat{\varphi})$ is an arbitrary smooth periodic function of the coordinate $\hat{\varphi}$. It is convenient to define $\epsilon_n(\hat{\varphi}) = -e^{-in\hat{\varphi}}$ and $\zeta_n = \zeta(\epsilon_n)$, where n are integers. Then the asymptotic symmetry group is generated by

$$
\zeta_n = -e^{-in\hat{\varphi}}\frac{\partial}{\partial\hat{\varphi}} - in\hat{r}e^{-in\hat{\varphi}}\frac{\partial}{\partial\hat{r}} \;,
\tag{13}
$$

which obey the Virasoro algebra

$$
i[\zeta_m, \zeta_n] = (m-n)\zeta_{m+n} \;.
\tag{14}
$$

Each diffeomorphism ζ_n is associated to a conserved charge defined by[4]

$$
Q_\zeta = \frac{1}{8\pi}\int_{\partial\Sigma} k_\zeta \;,
\tag{15}
$$

where $\partial\Sigma$ is a spatial slice, and 2-form k_ζ is defined as

$$
\begin{aligned}
k_\zeta[h, g] &= \frac{1}{2}\left[\zeta_\nu\nabla_\mu h - \zeta_\nu\nabla_\sigma h_\mu{}^\sigma + \zeta_\sigma\nabla_\nu h_\mu{}^\sigma + \frac{1}{2}h\nabla_\nu\zeta_\mu \right.\\
&\left. -h_\nu{}^\sigma\nabla_\sigma\zeta_\mu + \frac{1}{2}h_{\nu\sigma}\left(\nabla_\mu\zeta^\sigma + \nabla^\sigma\zeta_\mu\right) \right] * (dx^\mu \wedge dx^\nu) \;,
\end{aligned}
\tag{16}
$$

where $*$ denotes the Hodge dual. The Dirac brackets of the conserved charges are just the common forms of the Virasoro algebras with central terms

$$
\frac{1}{8\pi}\int_{\partial\Sigma} k_{\zeta_m}[\mathcal{L}_{\zeta_n}g, g] = -\frac{i}{12}c(m^3 + \alpha m)\delta_{m+n,0} \;,
\tag{17}
$$

where c denote the central charge corresponding to the diffeomorphism and α is a trial constant. Evaluating the integral for the case of the near-horizon extremal Kaluza-Klein metric, we find the central charge

$$c = \frac{12\mu a}{\sqrt{1 - \nu^2}} \ . \tag{18}$$

This result exactly agrees with the one obtained by Azeyanagi et al. in [6]. It should be noted that the central charge can also be written as $c = 12J$. This relation between the central charge and angular momentum is just the same as that for Kerr black hole[1] and other examples of the Extremal Kerr/CFT dual.

After obtaining the central charge of the extremal Kaluza-Klein black hole, we now begin to get its CFT entropy. To get this, we have to calculate the generalized temperature with respect to the Frolov-Thorne vacuum. We consider the quantum field with eigenmodes of the asymptotic energy ω and angular momentum m, which are given by the following form

$$e^{-i\omega t + im\varphi} = e^{-i\left(\omega - \frac{m\sqrt{1-\nu^2}}{2a}\right)\frac{r_0}{\epsilon}\hat{t} + im\hat{\varphi}} = e^{-in_R\hat{t} + in_L\hat{\varphi}} \ , \tag{19}$$

with

$$n_R = \left(\omega - \frac{m\sqrt{1 - \nu^2}}{2a}\right)\frac{r_0}{\epsilon} \ , \quad n_L = m \ . \tag{20}$$

The correspondence Boltzmann factor is of the form

$$e^{-\frac{\omega - m\Omega}{T_H}} = e^{-\frac{n_R}{T_R} - \frac{n_L}{T_L}} \ , \tag{21}$$

where the left and right temperatures are given by

$$\begin{aligned} T_R &= \frac{r_0}{\epsilon}T_H \ , \\ T_L &= \frac{T_H}{\frac{\sqrt{1-\nu^2}}{2a} - \Omega_H} \ . \end{aligned} \tag{22}$$

In the extremal limit $\mu \to a$, the left and right temperatures reduce to

$$T_R = 0 \ , \quad T_L = \frac{1}{2\pi} \ . \tag{23}$$

According to the Cardy formula the entropy for a unitary CFT, we can obtain the microscopic entropy of the extremal Kaluza-Klein black hole

$$S_{CFT} = \frac{\pi^2}{3}cT_L = \frac{2\pi\mu a}{1 - \nu^2} \ , \tag{24}$$

which precisely agrees with the Bekenstein-Hawking entropy.

IV. ENTROPY OF NON-EXTREMAL KALUZA-KLEIN BLACK HOLE FROM HIDDEN CONFORMAL SYMMETRY

In this section, we extend the analysis of hidden conformal symmetry for the Kerr black hole to the Kaluza-Klein balck hole.

Let us consider the Klein-Gordon equation for the neutral massless scalar field in the background of the Kaluza-Klein black hole

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right)\Phi = 0 \ . \tag{25}$$

The scalar field wave function can be expanded in eigenmodes as

$$\Phi = e^{-i\omega t + im\varphi}\Phi(r,\theta) \ , \tag{26}$$

where ω and m are the quantum numbers. Then the scalar field wave equation can be separated into the spheroidal equation and the radial equation

$$\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right) - \left(\frac{m^2}{\sin^2\theta} + a^2\omega^2\sin^2\theta\right)\right]S(\theta) = -\lambda S(\theta) \ ,$$

$$\left[\partial_r\Delta\partial_r + \frac{R^4(r)(\omega - m\Omega(r))^2}{\Delta} + \frac{1}{R^4(r)}m^2a^2\left(r^2 + a^2 + \frac{2\mu r}{1-\nu^2}\right)\right]\Psi = \lambda\Psi \ , \tag{27}$$

where λ is the separation constant and

$$\Omega(r) = \frac{2\mu r}{\sqrt{1-\nu^2}}\frac{a}{R^4(r)} \ ,$$

$$R^4(r) = (r^2 + a^2)\left(r^2 + a^2 + \frac{\nu^2}{1-\nu^2}2\mu r\right) \ . \tag{28}$$

The radial equation can also be rewritten as

$$\left[\partial_r\Delta\partial_r + \frac{\left(2\mu r_+\omega/\sqrt{1-\nu^2} - am\right)^2}{(r-r_+)(r_+ - r_-)} - \frac{\left(2\mu r_-\omega/\sqrt{1-\nu^2} - am\right)^2}{(r-r_-)(r_+ - r_-)}\right.$$

$$\left. + \left(r^2 + a^2 + \frac{2\mu(2\mu - r)}{1-\nu^2}\right)\omega^2\right]\Psi = \lambda\Psi \ . \tag{29}$$

Following the argument of CMS, we also consider the same near-region, which is defined by the condition

$$\omega\mu \ll 1 \ , \quad \omega r \ll 1 \ . \tag{30}$$

In the near-region, the angular equation reduces to the standard Laplacian on the 2-sphere with the separation constants taking values

$$\lambda = l(l+1) \ . \tag{31}$$

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And the radial equation can be simplified as

$$\left[\partial_r \Delta \partial_r + \frac{\left(2\mu r_+ \omega/\sqrt{1-\nu^2} - am\right)^2}{(r-r_+)(r_+-r_-)} - \frac{\left(2\mu r_- \omega/\sqrt{1-\nu^2} - am\right)^2}{(r-r_-)(r_+-r_-)}\right]\Psi = l(l+1)\Psi . \quad (32)$$

The above equation can be solved by hypergeometric functions. As hypergeometric functions transform in representations of SL(2,R), this suggests the existence of a hidden conformal symmetry. Now we will show that the radial equation can also be obtained by using of the SL(2,R) Casimir operator.

Introducing the coordinates

$$\begin{aligned}
w^+ &= \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R \varphi} , \\
w^- &= \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L \varphi - 2n_L t} , \\
y &= \sqrt{\frac{r_+-r_-}{r-r_-}}e^{\pi(T_L+T_R)\varphi - n_L t} ,
\end{aligned} \quad (33)$$

with

$$T_R = \frac{r_+-r_-}{4\pi a} , \quad T_L = \frac{r_++r_-}{4\pi a} , \quad n_L = \frac{\sqrt{1-\nu^2}}{4\mu} . \quad (34)$$

Then we can locally define the vector fields

$$\begin{aligned}
H_1 &= i\partial_+ , \\
H_0 &= i(w^+\partial_+ + \frac{1}{2}y\partial_y) , \\
H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) ,
\end{aligned} \quad (35)$$

and

$$\begin{aligned}
\bar{H}_1 &= i\partial_- , \\
\bar{H}_0 &= i(w^-\partial_- + \frac{1}{2}y\partial_y) , \\
\bar{H}_{-1} &= i(w^{-2}\partial_- + w^- y\partial_y - y^2\partial_+) ,
\end{aligned} \quad (36)$$

These vector fields obey the SL(2, R) Lie algebra

$$[H_0, H_{\pm 1}] = \mp i H_{\pm 1} , \quad [H_{-1}, H_1] = -2i H_0 , \quad (37)$$

and similarly for $(\bar{H}_0, \bar{H}_{\pm 1})$. The SL(2, R) quadratic Casimir operator is

$$\begin{aligned}
\mathcal{H}^2 = \bar{\mathcal{H}}^2 &= -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) \\
&= \frac{1}{4}(y^2\partial_y^2 - y\partial_y) + y^2\partial_+\partial_- .
\end{aligned} \quad (38)$$

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In terms of the (t, r, φ) coordinates, the SL(2, R) quadratic Casimir operator becomes

$$\mathcal{H}^2 = \partial_r \Delta \partial_r - \frac{\left(2\mu r_+ \partial_t / \sqrt{1 - \nu^2} + a \partial_\varphi\right)^2}{(r - r_+)(r_+ - r_-)} + \frac{\left(2\mu r_- \partial_t / \sqrt{1 - \nu^2} + a \partial_\varphi\right)^2}{(r - r_-)(r_+ - r_-)} . \tag{39}$$

So the near region wave equation can be written as

$$\mathcal{H}^2 \Phi = \bar{\mathcal{H}}^2 \Phi = l(l+1) \Phi , \tag{40}$$

and the conformal weights of dual operator of the massless field Φ should be

$$(h_L, h_R) = (l, l) . \tag{41}$$

Now, we want to calculate the microscopic entropy of the dual CFT, and compare it with the Bekenstein-Hawking entropy of the non-extremal Kaluza-Klein black hole. For the extremal case, the central charges can be derived from an analysis of the asymptotic symmetry group as we did in the last section. However, we did not know how to extend this calculation away from extremality. As did in [40], we will simply assume that the conformal symmetry found here connects smoothly to that of the extreme limit and the central charge still keeps the same as the extremal case, which is given by Eq.(18). The microscopic entropy of the dual CFT can be computed by the Cardy formula

$$S_{CFT} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = \frac{2\pi\mu}{\sqrt{1 - \nu^2}}(\mu + \sqrt{\mu^2 - a^2}) , \tag{42}$$

which matches with the black hole Bekenstein-Hawking entropy.

V. CONCLUSION

In this paper, we have extend the recently proposed Kerr/CFT correspondence to examine the dual conformal field theory of the Kaluza-Klein black hole. Firstly, for the extremal Kaluza-Klein black hole, we have calculated the central charge and temperature of the dual conformal field by employing the approach of GHSS. It is shown that the microscopic entropy calculated by using Cardy formula agrees with the Bekenstein-Hawking entropy of the extremal Kaluza-Klein black hole. Then, for the non-extremal case, we have investigated the hidden conformal symmetry of Kaluza-Klein black hole by studying the near-region wave equation of a neutral massless scalar field, and found the left and right temperatures of the

dual conformal field theory. Furthermore, the entropy of non-extremal Kaluza-Klein black hole is reproduced by using Cardy formula.

The results of this paper totally support the arguments made in Ref.[40], which suggests that, for the general rotating black hole, even away from extremality, there is a dual two dimensional conformal field theory with the left and right excited modes. Up to now, only the hidden conformal symmetry of the near-region scalar field wave equation is studied. It would also be interesting to investigate whether the hidden conformal symmetry can be obtained by studying the near-region wave equation of high spin field.

Acknowledgement

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